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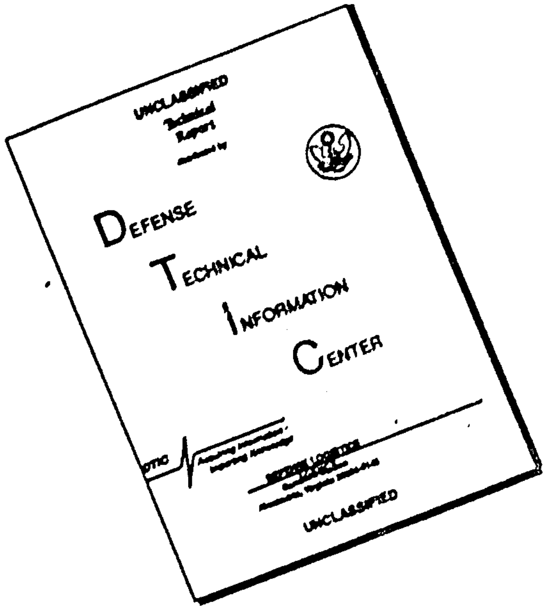

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RESISTANCE VARIATION OF EXPLODING WIRES

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MECHANICS SECTION

RESISTANCE VARIATION OF EXPLODING WIRES*

By

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ABSTRACT

When fine wires are exploded by discharging a capacitor, the circuit exhibits damped electrical oscillations as an L-R-C circuit; however, the resistance varies during the explosion which complicates the analysis. To date the resistance has been calculated either by integrating twice in succession the record of the rate of change of the current[1], or by the current-voltage method[2]. A different approach is proposed in which the curcuit equation is differentiated with respect to time and then solved by a method similar to the WKB method. The method will be discussed, original data provided, and values of calculated resistances obtained by the double integration method will be used to indicate the validity of the solution.

TABLE OF CONTENTS

I. INTRODUCTION

As evidenced by publications in the literature, conferences such as the present one, and excellent adaptations to industrial fields, experimenters have been successful in using exploding wires for many different tasks. They have done this with a minimum of theoretical work and a maximum of experimental effort. This work represents another attempt to combine empirical measurements with basic theory to determine whether a postulated resistance variation can fit the observed.data.

We shall solve the circuit equation with a constant resistance as a model; solve the same equation with a resistance that varies with time using a method similar to that used by Wenzel, Kamers, & Brillouin to indicate the algebraic form required for the resistance; use this form to establish the terms of a series solution for the current in the circuit; and lastly show how the solution can be made to fit the observed data.

II. MATHEMATICAL PROCEDURE

The basic circuit equation for the capacitor discharge circuit is (as usual)

$$L \frac{dI}{dt} + RI + \frac{1}{C} \int_0^t I dt = V_o \tag{1}$$

where L is the inductance of the circuit, I is the current in the circuit, t is the time measured from switch closure, R is the sum of the wire resistance and the resistance of the circuit elements, C is the capacitance of the circuit, and V_o is the voltage to which the capacitor was charged at switch closure time.

To solve this equation, it is first differentiated with respect to time to eliminate the integral. In this work the L, C, and V_0 are assumed to be independent of time.[4] The equation becomes

$$L \frac{d^2t}{dt^2} + R \frac{dI}{dt} + I \frac{dR}{dt} + \frac{1}{C} = 0 \tag{2}$$

A. Solution with R Constant

At the risk of repetition we will solve this equation with a constant resistance to form a limiting case for a later solution. One assumes a solution with two constants A and B which are evaluated with the initial conditions.

$$I = Ae^{wt} + Be^{w^* t} \tag{3}$$

w and w^* are taken to be complex conjugates with A the real part and B the imaginary part.

$$w = a + ib \tag{4}$$

The assumed solution is set into the differential equation. By setting the coefficients

$$Ae^{wt}(Lw^2 + Rw + \frac{1}{C}) + Be^{w^* t}(Lw^{*2} + Rw^* + \frac{1}{C}) = 0 \tag{5}$$

of the exponentials independently equal to zero one solves for a and b in terms of L, R, and C. This leads to

$$a = -\frac{R}{2L} \tag{6}$$

$$b = \pm\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{7}$$

Thus,

$$I = A \exp\left\{-\frac{Rt}{2L} \pm it\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\right\} + B \exp\left\{-\frac{Rt}{2L} \mp it\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\right\} \tag{8}$$

With the usual initial conditions that

$$I = \int_o^t Idt = 0 \tag{9a}$$

$$t = 0 \tag{9b}$$

the current is found to be

$$I = \frac{V_o \exp -\frac{Rt}{2L}}{L\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}} \sin t\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{10}$$

B. Solution with R a Variable in Time

The WKB method of solving this type of equation assumes that the variation of R is small with respect to itself and takes place in a time short compared with the basic period of oscillation. While R does vary during a small portion of the base wave length in the solution, it varies by a large amount; therefore, we shall not be able to follow the WKB method completely to simplify the differential equation. However, we shall use the general process to obtain an equation for solution.

As before, we assume a solution but allow the w's (which means the a's and the b's also) to be a function of time. In this case, with arbitraty constants C and D intended to be determined by the initial conditions, we have

$$I = C \exp\left[\int_0^t w\,dt\right] + D \exp\left[\int_0^t w^* dt\right] \tag{11}$$

in which the variation of w with time is inserted by employing the integral representation. This solution is substituted into the euqation to obtain

$$C \exp\left[\int_0^t w\,dt\right]\left(L\frac{dw}{dt} + L w^2 + \frac{dR}{dt} + Rw + \frac{1}{C}\right)$$

$$+ D \exp\left[\int_0^t w^* dt\right]\left(+ L\frac{dw^*}{dt} + L w^{*2} + \frac{dR}{dt} + R w^* + \frac{1}{C}\right) = 0 \tag{12}$$

As before, we substitute in the original equation with $w = a + ib$; set the coefficients of the exponential terms separately equal to zero; separate the real and imaginary parts of these equations and solve for a and b in terms of R, L. C and t. This process leads to

$$a = -\frac{1}{2b}\frac{db}{dt} - \frac{R}{2L} \qquad (13)$$

$$\frac{1}{2b}\frac{d^2b}{dt^2} - \frac{3}{4b^2}\left(\frac{db}{dt}\right)^2 + b^2 = \frac{\frac{dR}{dt}}{2L} - \frac{R^2}{4L^2} + \frac{1}{LC} \qquad (14)$$

The equation for b obtained by using $w^* = a - ib$ will be the same as Equation 14.

At this point in the WKB process, the second derivatives and products of first derivatives are assumed to be small enough to be neglected. In Equation 14 the solution for b would be at hand; however, given the solution for b in that case the term with the first derivative is no longer neglectable. If one includes this term and solves for b, he finds that the value of the second derivative term is now no longer neglectable. Therefore, no terms are neglectable in this case. The complementary solution of Equation 14 is found to be

$$b = \frac{i}{2(t+r+is)} - \frac{i}{2(t+r-is)} = \frac{s}{(t+r)^2 + s^2} \qquad (15)$$

in which r and s are the two undetermined constants required for a second order differential equation.

The particular solution of this equation depends upon the form of the right hand side since it is the driving function. Usually a solution is relatively easy to obtain if the driving function is of the same form as the complementary solution. Therefore, we let R be expressed in the form of a truncated series. Then, b may be obtained as a series of this same form that might hopefully be truncated without a great loss of accuracy. A series for R with terms similar to Equation 15 if s is suppressed in favor of r might be

$$R = k + \frac{h}{t+g} + \frac{g'}{(t+g)^2} + \frac{f}{(t+g)^3} + \frac{e}{(t+g)^4} \tag{16}$$

as an equation in which k, h, g', f, e and g are constants.[5] We assume accordingly a series solution for b

$$b = \sum_{j=0}^{s} \frac{m_j}{(t+n)^j} \tag{17}$$

set it into the differential equation, set the coefficients of $(t + n)^{-j}$ terms separately equal to zero and determine the coefficients m_j.

After a series of algebraic manipulations, b is found to be

$$\begin{aligned} b = W &+ \frac{S}{2W}\left(\frac{1}{t+g}\right) + \left(\frac{P}{2W} - \frac{S^2}{8W^3}\right)\left(\frac{1}{t+g}\right)^2 + \left(\frac{S^3}{16W^5} - \frac{S}{4W^3}\right. \\ &\left. - \frac{PS}{4W^3} + \frac{Q}{2W}\right)\left(\frac{1}{t+g}\right)^3 + \left(-\frac{3P}{4W^3} - \frac{P^2}{8W^3} + \frac{3PS^2}{16W^5} + \frac{17S^2}{32W^5}\right. \\ &\left. - \frac{5S^4}{128W^2} - \frac{SQ}{4W^3} + \frac{M}{2W}\right)\left(\frac{1}{t+g}\right)^4 + \ldots \end{aligned} \tag{18}$$

and a is determined as

$$a = -\frac{k}{2L} - \frac{h}{2L}\left(\frac{1}{t+g}\right) + \left(\frac{S}{2W^2} - \frac{g'}{2L}\right)\left(\frac{1}{t+g}\right)^2 - \left(\frac{S^2}{4W^4} - \frac{P}{2W^2} + \frac{f}{2L}\right)\left(\frac{1}{t+g}\right)^3$$

$$-\frac{1}{2}\left(\frac{3PS}{2W^4} + \frac{3S}{4W^4} - \frac{S^3}{2W^6} - \frac{3Q}{2W^2} + \frac{e}{L}\right)\left(\frac{1}{t+g}\right)^4 + \ldots \tag{19}$$

in which

$$W^2 = \frac{1}{LC} - \frac{k^2}{4L^2} \tag{20}$$

$$S = -2\left(\frac{h}{2L}\right)\left(\frac{k}{2L}\right) \tag{21}$$

$$Q = -\frac{g'}{L} - \left(\frac{h}{2L}\right)\left(\frac{g'}{L}\right) - \left(\frac{k}{2L}\right)\left(\frac{f}{L}\right) \tag{22}$$

$$M = -\left(\frac{g'}{2L}\right)^2 - \frac{3}{2}\,\frac{f}{L} - \left(\frac{h}{2L}\right)\left(\frac{f}{L}\right) - \left(\frac{k}{2L}\right)\left(\frac{e}{L}\right) \tag{23}$$

$$P = -\left(\frac{h}{2L}\right)\left(1 + \frac{h}{2L}\right) - \left(\frac{k}{2L}\right)\left(\frac{g'}{L}\right) \tag{24}$$

The first three terms of the exponent ($\int_0^t$ wdt) are given as a real part

$$E_r = -\frac{kt}{2L} - \frac{h}{2L}\,\ell n\,\frac{t+g}{g} - \left[\frac{\left(\frac{h}{2L}\right)\left(\frac{k}{2L}\right)}{2W^2} + \frac{g'}{2L}\right]\frac{t}{g(t+g)} - \ldots \tag{25}$$

and an imaginary part

$$E_i = t\sqrt{\frac{1}{LC} - \frac{k^2}{4L^2}} - \frac{\left(\frac{h}{2L}\right)\left(\frac{k}{2L}\right)}{W} \ln \frac{t+g}{g}$$

$$- \left[\frac{\frac{h}{2L}\left(1 + \frac{h}{2L}\right) + \left(\frac{k}{2L}\right)\left(\frac{g'}{L}\right)}{2W} + \frac{\left(\frac{h}{2L}\right)^2 \left(\frac{k}{2L}\right)^2}{2W^2}\right] \frac{t}{g(t+g)} - \ldots \tag{26}$$

Given the initial conditions and the terms E_r and E_i, one finds that the expression for the current is as could be shown in part A

$$I = \frac{V_o \exp\left[E_r\right]}{L\frac{dE_i}{dt}} \sin\left[E_i\right] \tag{27}$$

C. Comparison of Solutions

The following statements may be made about the solution for the constant and variable cases:

(1) The solution for the variable R case being a series should reduce to the constant R case in the limit. We find this to be so: when $h = g = f = e = 0$, E_r in Equation 25 reduces to Equation 6 and E_i reduces to Equation 7.

(2) From experiments, the current approaches a damped oscillation which implies fairly constant resistance. We find that R approaches a steady

value given by k as the time gets large. Looking at E_r, we find that the third (and succeeding although not shown) terms approach a steady value because $\frac{t}{t+g}$ approaches one. The second term continues to increase but not as rapidly as the first term. Thus, the curve decays at a slightly larger rate for the variable than for the constant resistance.

E_i exhibits the same behavior. However, since E_i is the argument of the sine function, the main effect is to produce a modulation of the phase or frequency. Thus, one would expect a large variation at the beginning of the discharge and slower one at the end of the discharge. These changes will cause the curve to be nonsinusoidal.

The amplitude factor contains dE_i/dt which is also a function of time. Initially the variation is large but will damp out because the time varying terms have t in their denominator. Thus, for long times we find that

$$\frac{dE_i}{dt} \text{ approaches } \sqrt{\frac{1}{LC} - \frac{k^2}{4L^2}}\,. \tag{28}$$

which is found in the solution for a constant R. (See Equation 10.)

III. EXPERIMENTAL DATA

The above expression for the current should be fitted to the data so that the various constants might be obtained. Because the various terms are made up of a series expansion such curve fitting would be quite complicated. In place of this method we shall use the double integration method to determine the value of R.

From the original circuit equation the resistance is given as shown in reference 1.

$$R = \frac{V_o - L\frac{dI}{dt} - \frac{1}{C}\int_o^t Idt}{I} \tag{29}$$

Since the rate of change of the current is measured with a Rogowsky loop, the current is obtained by integrating once and the integral of the current by integrating twice. With the calibration constant one can obtain the value of the resistance as shown in the equation. If one uses a current shunt, the other terms are obtained by differentiating and integrating the data for the current.

Since every experimenter has different equipment and since his results are largely a matter of his instrumentation, Figure 1 shows the exploding wire apparatus used to obtain our data. The 12 capacitors can be charged to 50 KV thereby having a storage capability of 10^4 joules. The ringing frequency is 275 kilocycles so that the pulses are made up of 3 or 4 cycles that carry the bulk of the power. The wire commonly used is 2 mil tungsten 1/2 inch long. Figure 2 shows the cartridge chamber housing the wire.

A Rogowsky loop measures the rate of change of the current as shown in Figure 3. The sweep speed of the Tektronix scope is 2 microseconds/cm which speed is used to obtain the values of L and R in the main part of the pulse. Figure 4 shows the pulse taken at 1/2 microsecond/cm. This shows the shape of the first half cycle. After reading the deflections with a 12X microscope, data from the curve is plotted as shown in Figure 5.

The frequency is determined from the cross over points thereby giving the inductance from the formula

$$\nu = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{30}$$

Using curves as shown in Figure 3 the value of R in the steady portion of the decay curve is found to be about 0.030 ohms. Using this value of resistance the inductance is found to 0.111×10^{-6} Henries. Using these numbers, numerical integration gives us

$$I = \int_0^t \frac{dI}{dt}\, dt \quad \text{and} \tag{31}$$

$$\int_0^t dt \int_0^t \frac{dI}{dt}\, dt = \int_0^t I dt \tag{32}$$

from which the circuit resistance is obtained for each interval of time. This relation is also plotted on Figure 5.

By standard curve fitting methods the equation for the resistance curve may be obtained, when t is expressed in microseconds and R in ohms.

$$R = 0.030 - \frac{0.0244}{t + 0.01} + \frac{0.048}{(t + 0.01)^2} - \frac{0.00650}{(t + 0.01)^3} + \frac{0.000271}{(t + 0.01)^4} \tag{33}$$

This shows that the resistance is approaching the limit 0.030 as time increases. It also indicates that many terms are needed to get an accurate value because

the coefficients do not decrease rapidly. This indicates that many terms will be needed in the expression for the current.

To obtain a possible rationale for the particular form of the equation for the resistance, one might propose the following derivation. According to Lin[6] (and Bennett[7]) the radius of the shock wave proceeding outward from a line explosion is proportional to the square root of the time since the explosion. If one thinks of the current path as being the material behind or within that shock wave, then the cross sectional area will be proportional to that radius squared. By the simplest theory of conductors the resistance is inversely proportional to the cross-sectional area. Therefore, the resistance of the circuit will have a constant term for the busbars and other items, and it will have a term for the wire or plasma or gap that is inversely proportional to a time interval. Since the resistance is also a function of the number of current carriers, their mean-free-path, and their temperature or internal energy, it is not surprising that other terms should be included in the series expression for the total resistance.

IV. CONCLUSIONS

(1) The circuit equation was solved by the series method. (2) A procedure was shown that indicated the type of series that should be used. (3) This series was found to be amenable to the data, but a series was obtained that converged rather slowly. (4) Terms were found in the series which would represent variations in frequency and/or phase.

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The author is grateful for the conversations held with Dr. J. F. Heyda on the mathematical derivations given above. The experimental techniques could not have been developed nor the data obtained without the efforts of Mr. C. J. Dudzinski of these laboratories.

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2. R. J. Reithel, et. al., 2nd Conference on Exploding Wire Phenomenon, Boston, Mass., Nov. 1961.

3. G. Wenzel, A Generalization of the Quantum Conditions for the Purposes of Wave Mechanics, Zeit f Physik, Vol. 38 (1926) pg. 518.

4. The L is not independent of time because the inductance of the wire depends upon its diameter and geometrical relation to the rest of the apparatus. The variation may be included in the solution if desired, although the work reported here neglects that variation. It should be noted in support of this assumption that the variation amounts to approximately 5% of the total inductance where the variation of resistance amounts to 100-1000%. Thus, we have chosen to study the resistance variation as the more important.

5. This expression best suits the resistance as the explosion process approaches a steady damped oscillation. The resistances as the wire is changing state and at the end of the explosion process are other problems.

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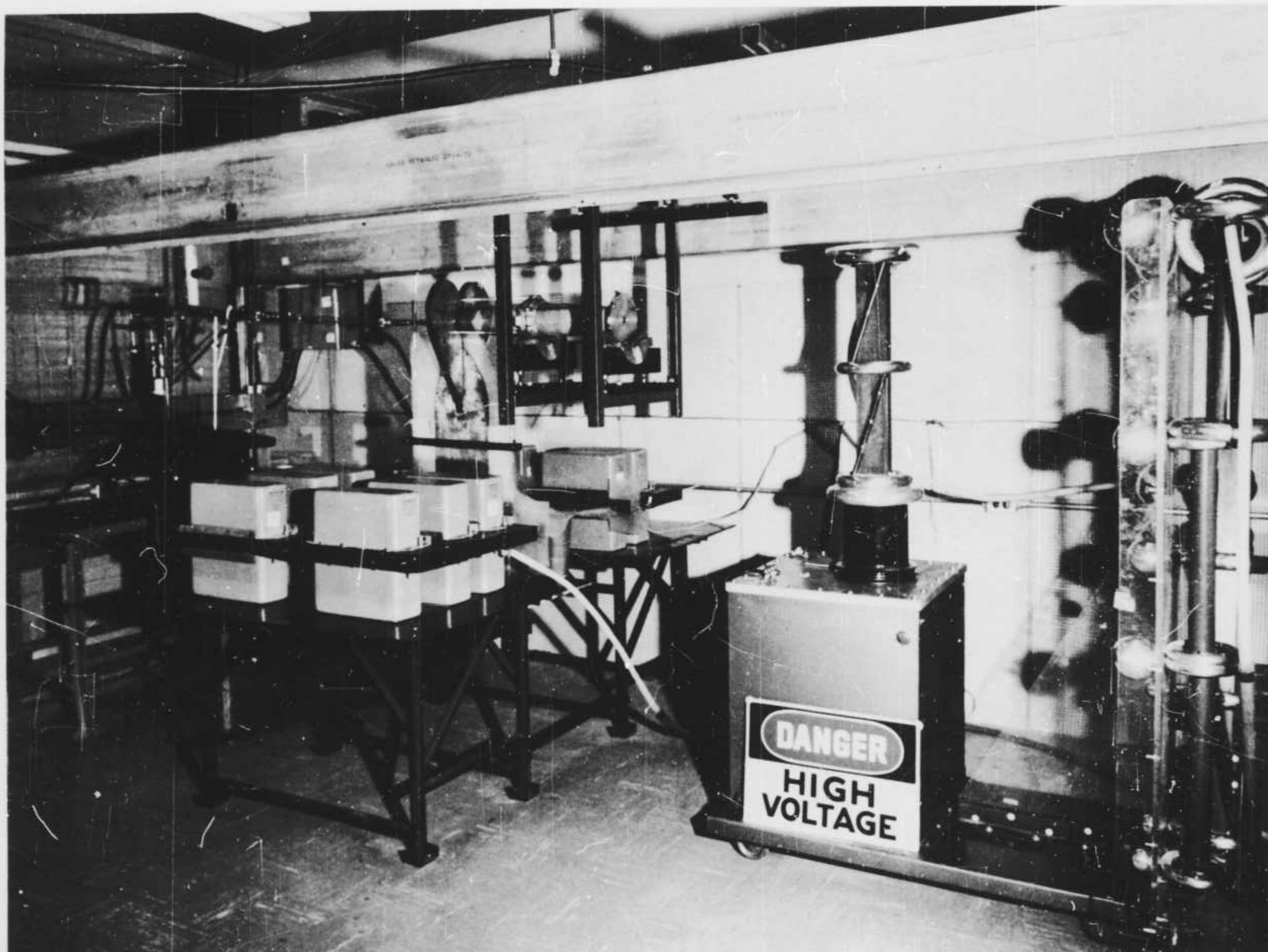

FIGURE 1. THERMAL IMPACT FACILITY

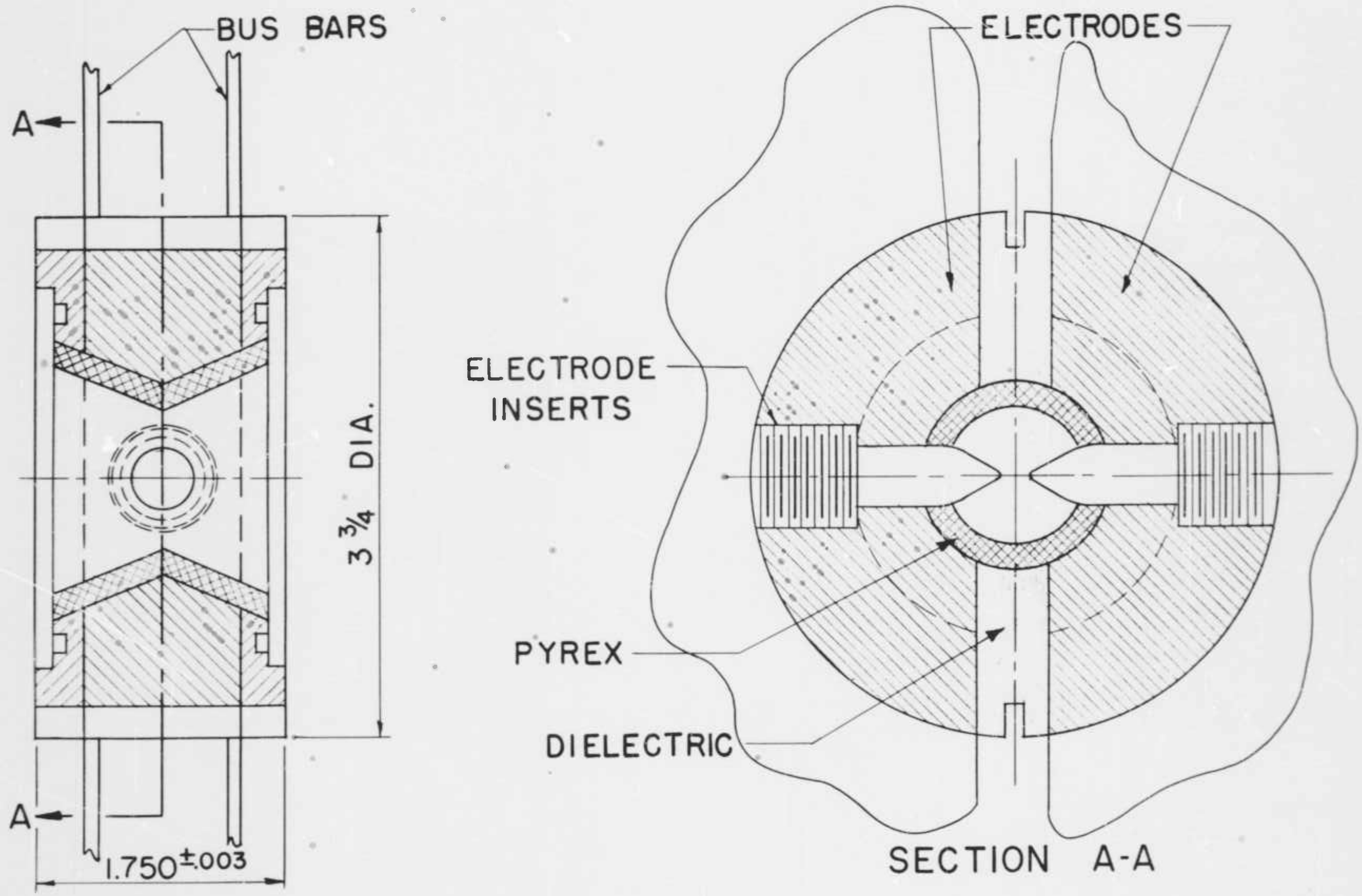

FIGURE 2. DISCHARGE CARTRIDGE FOR EXPLODING WIRE

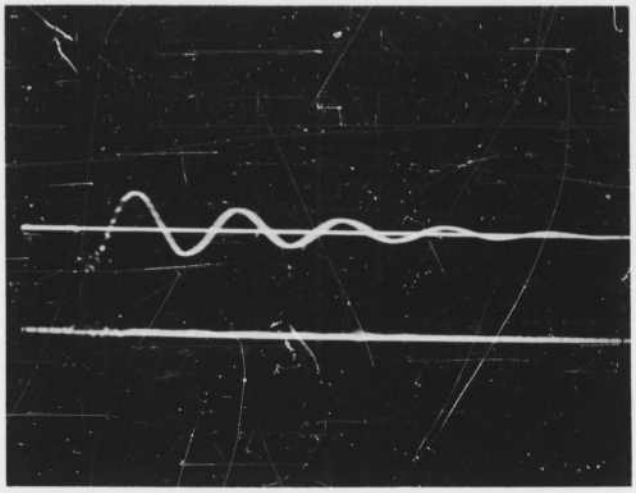

FIGURE 3. DECAY CURVE FOR ROGOWSKY CURRENT LOOP.

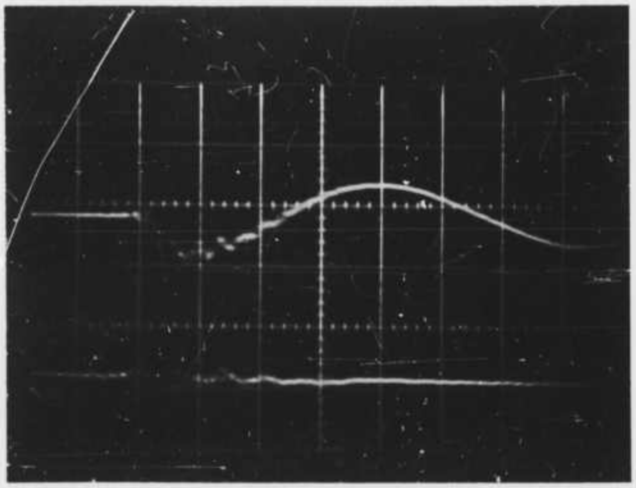

FIGURE 4. DECAY CURVE AT FASTER SWEEP SPEEDS

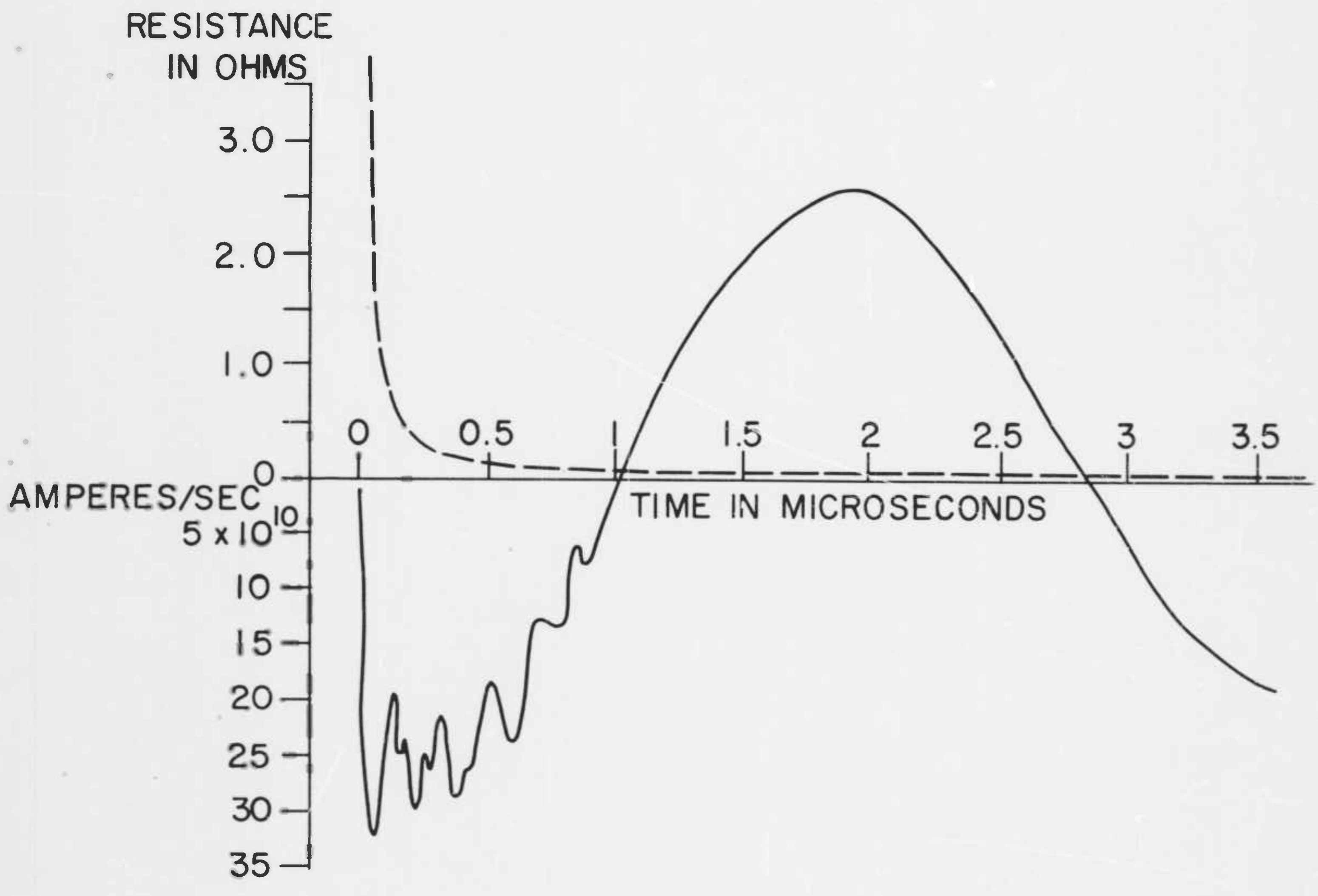

FIGURE 5. DI/DT DATA MEASUREMENTS AND RESISTANCE CALCULATIONS

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SUMMARY

When fine wires are exploded by discharging a capacitor, the circuit exhibits damped electrical oscillations as an L-R-C circuit; however, the resistance varies during the explosion which complicates the analysis. To date the resistance has been calculated either by integrating twice in succession the record of the rate of change of the current, or by the current-voltage method. A different approach is proposed in which the circuit equation is differentiated with respect to time and then solved by a me method similar to the WKB method. The method is discussed, original data provided, and values of calculated resistances obtained by the double integration method are used to indicate the validity of the solution.

KEY WORDS

Exploding wires, Resistance, WKB method

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